

An Alternative to Matter Localization in the “Brane World”: An Early Proposal and its Later Improvements

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Here we place the Latex typeset of the paper¹

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In the paper we presented the picture that our spacetime is a 3-brane moving in a higher dimensional space. The dynamical equations were derived from the action which is just that for the usual Dirac–Nambu–Goto p -brane. We also considered the case where not only one, but many branes of various dimensionalities are present, and showed that their intersections with the 3-brane manifest as matter in 4-dimensional spacetime. We considered a particular case, where the intersections behaved as point particles, and found out that they follow the geodesics on the 3-brane worldsheet (identified with our spacetime). In a series of subsequent papers the original idea has been further improved and developped. This is discussed in a note at the end, where it is also pointed out that such a model resolves the problem of massive matter confinement on the brane, recently discussed by Rubakov et al. and Mück et al. .

¹This rather old paper is not available elsewhere on the Internet. Therefore, facing the explosion of the activity concerning the brane world, I would like to make the paper easily accessible. The body of the paper is identical to the published version; a footnote is added, and a note at the end.

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Einstein's Gravity from a First Order Lagrangian in an Embedding Space

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We formulate a first order action principle in a higher dimensional space M_N in which we embed spacetime. The action I is essentially an “area” of a four-dimensional spacetime V_4 weighted with a matter density ω in M_N . For a suitably chosen ω we obtain on V_4 a set of worldlines. It is shown that these worldlines are geodesics of V_4 , provided that V_4 is a solution to our variational procedure. Then it follows that our spacetime satisfies the Einstein equations for dust – apart from an additional term with zero covariant divergence. (This extra term was shown in a previous paper to be exactly zero at least in the case of the cosmological dust model.) Thus we establish a remarkable connection of the extrinsic spacetime theory with the intrinsic general relativity. This step appears to be important for quantum gravity.

The idea of formulating a theory of gravity by using an embedding space has been issued by some authors such as Fronsdaal [1] and others [2]. So far it has not received much attention. A possible reason is that the Einstein general relativity based on the intrinsic geometry of spacetime is so successful. However, if we try to quantize the theory we face serious difficulties [3] which are only partially circumvented by the brilliant works of many authors [4]–[6]. Therefore it seems reasonable to search for extensions and/or alternatives to the Einstein gravity. A promising approach appears to be a reformulation of general relativity in the extrinsic instead of the intrinsic terms only.

In the previous papers [7, 8] we started to develop an idea which is briefly the following. We assume that the arena, in which the physical events (parametrized by the coordinates η^a , $a = 1, 2, \dots, N$) are situated, is a certain higher dimensional pseudoeuclidean space M_N – with a given dimension N (say 10). With these events we associate a field $\omega(\eta)$ which represents what I call *the matter density* in M_N . This higher dimensional world is “frozen” and motionless: everything is written in M_N once and for all.

Then we assume that somewhere in M_N the matter density is “wired” in such a way that it allows for the existence of *an observer*². By assumption, an observer registers (in classical approximation), at each step, only the events on a three-dimensional surface Σ ; at the next step on some other three-surface Σ' , etc.³ So the observer by the very act of successive observations introduces *the motion* of his simultaneity three-surface Σ through the higher space M_N . The set of all three-surfaces Σ belonging to the series of observations of our observer forms a four-dimensional continuum V_4 , called *spacetime*⁴, parametrized by the coordinates x^μ ($\mu = 0, 1, 2, 3$).

²Our observer by definition possesses the property called “*consciousness*” which in my terminology means that he not only “registers” the outside events, but he registers also the fact that he has registered, etc. ad “infinitem”. So we have a succession of registrations, and memory of what has been registered, and so on. Each registration together with the registration of registration, etc., we shall name *observation*. The succession of observations could be named “stream of consciousness”.

³**Note added in January, 2001:** This has to be taken with care. At a given moment of his proper time the observer, of course, does not register all the events on a surface Σ . Because of the finite speed of light, he is only able to accumulate data on Σ with the passage of his proper time and thus gradually expand the region of Σ known to him. Hence, he can talk about “motion” of the (simultaneity) surface Σ , but the latter is becoming known to him a posteriori. See also the note at the end.

⁴Since within the same wired structure \mathcal{B} in M_N supporting the existence of an observer \mathcal{O} we could start from various three-surfaces Σ which when moving would describe various spacetimes V_4 , we must provide a mechanism which would pay attention to one particular V_4 only. This is achieved through *the process of successive observations* – *picturesquely called “stream of consciousness”* – as defined in footnote 1. Various streams of consciousness are possible with the same structure \mathcal{B} and they belong to various observers \mathcal{O} on different spacetimes “going” through \mathcal{B} . The registrations alone as performed for instance by a measuring apparatus and stored in a magnetic tape) are not sufficient to

A spacetime V_4 is represented by the equation $\eta^a = \eta^a(x)$, whilst the induced metric tensor on V_4 is $g_{\mu\nu} = \partial_\mu \eta^a \partial_\nu \eta_a$. In a classical theory we assume that dynamically allowed are only such V_4 's which are solutions of a certain variational principle in M_N . In refs. [7, 8] we have taken the second order Einstein–Hilbert action I_H expressed in terms of the extrinsic variables η^a , $\partial_\mu \eta^a$, $\partial_\mu \partial_\nu \eta^a$ plus a corresponding matter action I_m . Varying $I_2 = I_H + I_m$ with respect to η^a we have obtained the covariant divergence of Einstein's equations multiplied by $\partial_\nu \eta_a$. These equations imply the validity of Einstein's equations with an additional term $C^{\mu\nu}(x)$, such that $(C^{\mu\nu} \partial_\nu \eta_a)_{;\mu} = 0$ and $C^{\mu\nu}_{;\nu} = 0$. Then we argued that in order to fix a solution (i.e., V_4) we need some additional equation (besides a choice of gauge). We have chosen the equation that results by varying the first order action⁵

$$I = \int \omega \sqrt{-g} d^4x = I[\eta^a(x)], \quad (1)$$

where ω is a function of position η^a in the embedding space and g the determinant of the intrinsic metric. Finally we have concluded that our starting action for the “field” $\eta^a(x)$ is I of eq. (1), whilst the equation resulting from the second order action I_2 served served for the purposes of calculating the four-velocity u^μ . In the present work I am going to demonstrate that we can avoid the use of I_2 and that the action (1) – for a suitable ω – already contains the Einstein equations for dust, apart from a function $C^{\mu\nu}(x)$ satisfying $C^{\mu\nu}_{;\nu} = 0$.

If we vary $\eta^a(x)$ in the action (1) we obtain [7, 8] (see also [9])

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} \omega \partial^\mu \eta_a) = \partial_a \omega \quad ; \quad \partial_a \equiv \frac{\partial}{\partial \eta^a}. \quad (2)$$

For a given $\omega(\eta)$, initial and boundary conditions and a chosen parametrization x^μ , we obtain a unique solution $\eta^a(x)$ describing a spacetime V_4 .

The matter density on a particular spacetime V_4 (represented by $\eta^a = \eta^a(x)$) is

$$\omega(\eta(x)) \equiv \rho(x) \quad (3)$$

and depends on the choice of $\omega(\eta)$ and of $\eta^a(x)$.

In the following we shall assume that the matter density $\omega(\eta)$ consists of massive m -dimensional sheets $\hat{V}_m^{(i)}$, parametrized by \hat{x}^A ($A = 1, 2, \dots, m$) and represented by the equation $\eta^a = \hat{\eta}_i^a(\hat{x})$; to this we also add a constant ω_0 :

$$\omega(\eta) = \omega_0 + \int \sum_i m_i \delta^N(\eta - \hat{\eta}_i) d^m \hat{x}. \quad (4)$$

determine a particular V_4 , that is a path of a three-surface Σ through M_N .

⁵**Note added in January, 2001 :** Such an action can be straightforwardly derived from the Dirac–Nambu–Goto action in a conformally flat embedding space [1].

From the point of view intrinsic to a particular V_4 (already given as a solution of eq. (2)), the matter density $\omega(\eta)$ of eq. (4) becomes (up to ω_0) that of point particles:

$$\omega(\eta(x)) \equiv \rho(x) = \omega_0 + \int \sum_i m_i \frac{1}{\sqrt{-g}} \delta^4(x - z_i) ds_i, \quad (5)$$

provided that we choose $m = N - 4 + 1$ (for $N = 10$ we have $m = 7$). That is, the intersection $V_4 \cap \bigcup_i \hat{V}_m^{(i)}$ gives (5). In other words, when crossing V_N with a spacetime V_4 , then, using (4) we obtain on V_4 the set of worldlines $x^\mu = z_i^\mu(\lambda)$; on a three-surface Σ this is the matter distribution of point particles, forming a so called *dust*.

Inserting (5) into (1) we have, apart from the term $I_0 = \int \omega_0 \sqrt{-g} d^4x$, the action of point particles in a given gravitational field $g_{\mu\nu}$:

$$I = I_0 + \int \sum_i m_i ds_i = I_0 + \int \sum_i m_i (g_{\mu\nu} \dot{z}_i^\mu \dot{z}_i^\nu)^{1/2} d\lambda_i, \quad (6)$$

where ds_i is a proper time and λ_i and arbitrary parameter along the i th worldline.

It is really remarkable that the gravitational field $g_{\mu\nu}(x)$ in *the intrinsic* action (6) is actually determined by the variation of the original action in V_N with respect to $\eta^a(x)$ (eqs. (1) and (4)). Usually – when starting from the intrinsic point of view – the action (5) describes (dust) particles in a *fixed* field $g_{\mu\nu}(x)$; varying z_i^μ we obtain the geodesic equation. To obtain the equations of motion for $g_{\mu\nu}$ we need an additional term, namely $(16\pi G)^{-1} \int \sqrt{-g} d^4x R$; this leads to the Einstein equations $G^{\mu\nu} = -8\pi G \rho u^\mu u^\nu$ which – because of the contracted Bianchi identity $(\rho u^\mu u^\nu)_{;\nu} = 0$ and the relation $(\rho u^\nu)_{;\nu} = 0$ for dust – imply the geodesic equation $u^\mu{}_{;\nu} u^\nu = 0$.

Now, from *the extrinsic point of view* (extrinsic relative to V_4) no additional terms are needed: the action (1) together with a properly chosen $\omega(\eta)$, in our case that of eq. (4), suffices both for the determination of a spacetime V_4 and a “motion”⁶ of particles in it.

However, it is not obvious that the equations for $\eta^a(x)$ (eq. (2)) contain the geodesic equation for a worldline. This I shall now explicitly demonstrate.

Let us integrate (2) over $\sqrt{-g} d^4x$ on a chosen V_4 and let the domain of integration Ω embrace one worldline only. Then we have

$$\int_\Omega \partial_\mu (\sqrt{-g} \omega \partial^\mu \eta_a) d^4x = \oint_B \sqrt{-g} \omega \partial^\mu \eta_a d\Sigma_\mu = \int_\Omega \partial_a \omega \sqrt{-g} d^4x, \quad (7)$$

⁶In our philosophy an observer’s stream of consciousness (see footnote 2) is actually moving (this implies that a spacelike three-surface Σ is moving), whilst matter is motionless. This Σ -motion leads to the illusion that point particles are moving in three-space. In four-space V_4 we have worldlines and point particles are three sections of worldlines; in still higher space M_N matter is represented by $m = N - 3$ sheets \hat{V}_m and worldlines are intersections with a chosen spacetime V_4 .

where $d\Sigma_\mu$ is an element of a three-surface B . The integration over the constant part of $\omega = \omega_0 + \omega_1$ can be made arbitrarily small, since for a point worldline we can shrink Ω to zero.

The boundary B in (7) consists of two spacelike three-surfaces Σ_1 and Σ_2 and a timelike three-surface; integration over the last one gives zero, since ω_1 is zero around a worldline. Then, writing $d\Sigma_\mu = n_\mu d\Sigma$, $U_a \equiv \partial^\mu \eta_a n_\mu$ and $d^4x = d\Sigma ds$ we have from (7)

$$\int_{\Sigma_1}^{\Sigma_2} \sqrt{-g} \omega_1 U_a d\Sigma = m_i U_a|_{\Sigma_2} - m_i U_a|_{\Sigma_1} = \int \partial_a \omega \sqrt{-g} d\Sigma ds_i. \quad (8)$$

If Σ_1 and Σ_2 are infinitesimally close, then (8) becomes

$$m_i \frac{dU_a}{ds_i} = \frac{d}{ds_i} \int \partial_a \omega_1 \sqrt{-g} d^4x = \int \partial_a \omega_1 \sqrt{-g} d\Sigma, \quad (9)$$

where m_i is the i th particle mass, n_μ the normal to Σ , and ds_i the element of proper time along the i th worldline.

Suppose that we have solved eq. (2) and found $\eta^a(x)$ for the case of $\omega(\eta)$ such as given in (4). The four-surface $\eta^a(x)$ ⁷ near the i th matter sheet $\hat{V}_7^{(i)}$ is affected by the presence of all the sheets \hat{V}_7 including one of the i th particle. A matter sheet \hat{V}_7 is supposed to influence V_4 in two ways

- i) it “twists” V_4 *without changing* (up to a reparametrization) *the intrinsic metric* $g_{\mu\nu}(x)$ in the vicinity of a worldline C_i ;
- ii) it additionally reshapes V_4 so that $g_{\mu\nu}$ (and curvature) close enough to C_i is also changed.

We repeat that a worldline C is the intersection of a spacetime V_4 and a matter sheet \hat{V}_7 . Different spacetimes give different worldlines C for the same matter sheet \hat{V}_7 . These different V_4 may all have the same intrinsic metric $g_{\mu\nu}(x)$ (and the curvature) everywhere except sufficiently close to C . Let us therefore write $\eta^a(x)$ as the sum of $\tilde{\eta}^a(x)$ due to the effect (i) and $\eta^{*a}(x)$ due to (ii):

$$\begin{aligned} \eta^a(x) &= \tilde{\eta}^a(x) + \eta^{*a}(x) \\ \partial_\mu \eta^a &= \partial_\mu \tilde{\eta}^a + \partial_\mu \eta^{*a}. \end{aligned} \quad (10)$$

Then we have

$$U^a = n^\mu \partial_\mu \eta^a = n^\mu (\partial_\mu \tilde{\eta}^a + \partial_\mu \eta^{*a}) \equiv \tilde{U}^a + U^{*a}. \quad (11)$$

⁷Here $\eta^a(x)$ is a representative of the class $\{\eta^a(x)\}$ of η^a ’s which all describe the same spacetime V_4 using different parametrizations.

The derivative ∂_μ can be split into the normal (relative to a spacelike three-surface Σ) derivative $\hat{\partial} \equiv n^\mu \partial_\mu$ and the tangential derivative $\bar{\partial}_\mu$ [10]. Then

$$U^{*a} \equiv n^\mu \partial_\mu \eta^{*a} = n^\mu (n_\mu \hat{\partial} \eta^{*a} + \bar{\partial}_\mu \eta^{*a}) = \hat{\partial} \eta^{*a}. \quad (12)$$

Since we can choose Σ so that on a worldline C the normal n^μ coincides with the tangent u^μ of C , it holds that $\hat{\partial} \eta^{*a}$ points into the direction of the worldline. Since worldline's density is the same everywhere on C , it follows from the symmetry consideration that $\eta^{*a}(x)$ is constant along a worldline and therefore $\hat{\partial} \eta^{*a} = 0$. Hence

$$U^a = \hat{U}^a \equiv n^\mu \partial_\mu \tilde{\eta}^a. \quad (13)$$

Let us multiply (9) by $\partial^\mu \tilde{\eta}_a$. Since in $\tilde{\eta}^a(x)$ we exclude the contribution of the i th particle to $g_{\mu\nu}$ (though we still retain the influence of the i th $\hat{V}_7^{(i)}$ to the extrinsic reshaping of V_4) it holds that $\partial^\mu \tilde{\eta}^a$ remains practically constant over the small four-region Ω surrounding our worldline. Therefore we have from (9)

$$\begin{aligned} m_i \frac{dU_a}{ds_i} \partial^\mu \tilde{\eta}^a &= \partial^\mu \tilde{\eta}^a \int \partial_a \omega_1 \sqrt{-g} d\Sigma \\ &\approx \int \partial^\mu \tilde{\eta}^a \partial_a \omega_1 \sqrt{-g} d\Sigma \\ &= \int \partial^\mu \omega_1 \sqrt{-g} d\Sigma \end{aligned} \quad (14)$$

Since [7]–[9] $U_a = \tilde{U}_a = \partial_\nu \tilde{\eta}_a e^\nu$, $\partial_\alpha \partial_\beta \tilde{\eta}_a \partial^\rho \tilde{\eta}^a = \tilde{\Gamma}_{\alpha\beta}^\rho$ and $\partial^\mu \tilde{\eta}^a \partial_\nu \tilde{\eta}_a = \delta^\mu_\nu$, where $\tilde{\Gamma}_{\alpha\beta}^\rho$ is the affinity of V_4 (described by $\tilde{\eta}^a(x)$) and u^ν is n^ν on the worldline, it is

$$\frac{dU_a}{ds} \partial^\mu \tilde{\eta}^a = \frac{du}{ds} + \tilde{\Gamma}_{\alpha\beta}^\mu u^\alpha u^\beta. \quad (15)$$

For the purpose of calculating the right-hand side of eq. (14) it is convenient to replace the δ -function in ω_1 by its finite, analogue, e.g., a normalized Gaussian function of width σ ; the δ -function is then a limit for $\sigma \rightarrow 0$.

Let us separate

$$\partial^\mu \omega_1 = n^\mu \hat{\partial} \omega_1 + \bar{\partial}^\mu \omega_1. \quad (16)$$

Since a worldline is the same at all values of proper time s , it follows that the derivative of ω_1 along a worldline is zero: $\hat{\partial} \omega_1 = 0$. The second term in (16) can be written in a special coordinate system as $\partial^r \omega_1$ with $r = 1, 2, 3$. The right-hand side of (14) is then $\int \partial^r \omega_1 \sqrt{-g} d\Sigma$ and it obviously vanishes if the domain of integration encloses a particle.

To be specific we now assume that

$$\omega_1(\eta(x)) \equiv \rho(x) - \omega_0 = \int \sum_i m_i A(\sigma) \exp \left[-\frac{(x - z_i)^2}{2\sigma^2} \right] ds_i, \quad (17)$$

where $A(\sigma)$ is a suitable normalization constant. In the limit $\sigma \rightarrow 0$ this becomes the sum of the δ -functions as given in (5). Using (17) the integral (14) becomes

$$\frac{d}{ds_i} \int_{\Omega_i} \partial^\mu \omega_1 \sqrt{-g} d^4x = \frac{d}{ds_i} \int_{\Omega_i} m_i A(\sigma) (x^\mu - z_i^\mu) \exp \left[-\frac{(x - z_i)^2}{2\sigma^2} \right] \sqrt{-g} d^4x = 0. \quad (18)$$

where the integration runs over a four-region Ω_i surrounding a point z_i^μ on a worldline C_i .

Therefore, eq. (14), which we obtained from the equation for $\eta^a(x)$ resulting from our first order variational principle (1), becomes the geodesic equation:

$$\frac{du}{ds} + \Gamma_{\alpha\beta}^\mu u^\alpha u^\beta = u^\mu{}_{;\nu} u^\nu = 0, \quad (19)$$

where we omit “ \sim ” which distinguishes V_4 from \tilde{V}_4 , since both spacetimes coincide everywhere except sufficiently near the worldline.

The result (19) is fascinating, because the same result also follows from Einstein’s equations for dust. Is the opposite also true, namely that (2) and (4) which lead to (19) imply Einstein’s equations for an induced metric?

Since in our case we also have $(\rho u^\nu)_{;\nu} = 0$, it follows from (19) that

$$(\rho u^\mu u^\nu)_{;\nu} = 0. \quad (20)$$

On the other hand our metric identically satisfies the contracted Bianchi identities

$$G^{\mu\nu}{}_{;\nu} = 0, \quad (21)$$

where $G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$ is the Einstein tensor. Eqs. (20) and (21) imply the existence of a function $C^{\mu\nu}(x)$ satisfying

$$G^{\mu\nu} = -8\pi G(\rho u^\mu u^\nu + C^{\mu\nu}) = -8\pi G T^{\mu\nu}, \quad (22)$$

where G is a suitable constant, e.g., the gravitational constant. Remember that in our case $G^{\mu\nu}$, ρ and u^μ are already given by a solution $\eta^a(x)$ of eqs. (2) and (4). Therefore eq. (22) is a defining equation for $C^{\mu\nu}(x)$. It measures a deviation of the proposed dust model (eqs. (2) and (4)) from Einstein’s equations for dust: the stress energy tensor $T^{\mu\nu}$ has the term for dust and the additional term $C^{\mu\nu}$. Though the induced metric and the four-velocity obtained from eqs (2) and (4) in general is not expected to satisfy the Einstein equations for dust, it is nevertheless significant that the deviative term $C^{\mu\nu}$ has zero covariant divergence and – as we have seen – the dust particles follow geodesics. In a previous work I have shown that there exists at least one case (namely the cosmological dust model) in which $C^{\mu\nu}$ is exactly zero.

It is really fascinating and important also for quantization that such result follows from the simple first order action (1). However, for the sake of completeness, it would be necessary to solve explicitly eq. (2) for $\eta^a(x)$, find the intrinsic metric in the vicinity of a point particle and compare it with the Schwarzschild solution of the Einstein equations. Analogous has been already done for a *continuous* ω [7, 8], and found that the situation is exactly as predicted by the cosmological dust solution of the Einstein equations.

Here we have considered, as a particular case, the intersection – in M_{10} – of V_4 with \hat{V}_7 which, in general, gives a worldline on V_4 . If instead of \hat{V}_7 we take \hat{V}_8 , then we obtain a worldsheet on V_4 , that is a *string* in three-space. It would be interesting to investigate also into this direction, especially in view of recent successes in string theory.

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Note added in January, 2001

In a series of papers [2]–[7] the model has been further elaborated. In ref. [3] a more direct proof that the worldline resulting from the intersection of two branes is a geodesic on the brane worldsheet has been found. In [2, 3] it was pointed out, following the known results of refs. [8], that the effective (4-dimensional) gravity on the brane is that of Einstein plus the higher order corrections.

In ref. [2]–[4] a conceptual shift has been gradually done, culminating in ref. [5], so that – in order to make the idea more along the lines with the current practice in physics – I did no longer worry about the observer, observation, succession of observations, etc.. Instead, I considered simply the extended objects, p -branes, living in spacetime which – according to the usual theory – has necessarily more than 4 dimensions. Then I proposed that, instead of trying to compactify those extra dimensions, it seem more naturally to assume that a 3-brane worldsheet V_4 already represents our spacetime. No compactification of the embedding space (also called target space) is necessary. In a recent preprint [7] I considered the case where many branes were present. All the branes were considered as dynamical, their equations of motion being derived from the corresponding action. The branes may intersect or self-intersect. It was shown that *the brane intersections and self-intersections behave as matter in the brane world*. This resolves the problem of matter confinement on the brane as pointed out recently by Mück, et al. [9], who found that massive particles cannot stably move along the brane: the brane is repulsive, and matter will be expelled from the brane into the extra dimensions. A similar behavior was observed by Dubovsky and Rubakov [10] in a different context.

In refs. [2]–[4] various technical and conceptual aspects⁸ of the idea that our world is a 3-brane living in a higher dimensional space were investigated, including the possibility of resolving the notorious “problem of time” in quantum gravity. A similar resolution, though not within the brane world context, has been proposed in ref. [11].

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⁸In those papers the issue raised in footnote 3 is elaborated further.

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